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W-3318(A)

M.A./M.Sc. (Fourth Semester) Examination, (Second Chance)

June-2020

MATHEMATICS

Paper - 412

Special Functions

Time : Three Hours

Maximum Marks : 85 ***Minimum Pass Marks : 29***

Note : Attempt **all** questions.

Unit-I

Q.1. a) Define Beta function. Obtain a relation between Beta and Gamma function.

b) Show that $\beta(m,m)=2^{1-2m}\beta\left(m,\frac{1}{2}\right)$

Unit-II

Q.2. If n is a non-negative integer and if a and b are independent of n, then prove that.

$$ {}_3F_1\left[\begin{matrix}-n,\ a+n,\ \frac{1}{2}+\frac{1}{2}a-b; \\ 1+a-b,\ \frac{1}{2}a+\frac{1}{2};\end{matrix}\ 1\right] $$

$$=\frac{(b)_n}{(1+a-b)_n}$$

Unit-III

Q.3. a) State and prove Rodrigue's formula for Legendre's polynomial.

b) Prove that

$$(2n+1)xp_n(x)=(n+1)P_{n+1}(x)+nP_{n-1}(x)$$

Unit-IV

Q.4. a) Prove that

$$\int_0^\infty e^{-x}L_n(x)L_m(x)\,dx=\begin{Bmatrix}0\,if\ m\neq n\\ 1\,if\ m=n\end{Bmatrix}$$

b) Prove that

$$L_n(0)=1$$

Unit-V

Q.5. a) Show that

$$\int_0^\infty e^{-y}y^{-\alpha}G_{p,q}^{m,n}\left(xy\Big/{}^{\alpha_1,\ \ldots\ldots,\alpha p}_{b_1,\ \ldots\ldots,bq}\right)dy$$

$$=G_{p+1,q}^{m,n+1}\left(x\Big/{}^{\alpha,\alpha_1,\ \ldots\ldots,\alpha p}_{b_1,\ldots\ldots,bq}\right)$$

b) Prove that

$$G_{p,q}^{m,n}\left(\frac{1}{x}\Big/{}^{ar}_{bs}\right)=G_{q,p}^{n,m}\left(x\Big/{}^{1-bs}_{1-ar}\right)$$